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# Spatial Visualization in Physics Problem Solving

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## Abstract

Three studies were conducted to examine the relation of spatial visualization to solving kinematics problems that involved either predicting the two-dimensional motion of an object, translating from one frame of reference to another, or interpreting kinematics graphs. In Study 1, 60 physics-naïve students were administered kinematics problems and spatial visualization ability tests. In Study 2, 17 (8 high- and 9 low-spatial ability) additional students completed think-aloud protocols while they solved the kinematics problems. In Study 3, the eye movements of fifteen (9 high- and 6 low-spatial ability) students were recorded while the students solved kinematics problems. In contrast to high-spatial students, most low-spatial students did not combine two motion vectors, were unable to switch frames of reference, and tended to interpret graphs literally. The results of the study suggest an important relationship between spatial visualization ability and solving kinematics problems with multiple spatial parameters.

*Keywords:* Spatial visualization ability; Problem solving; Physics; Visual-spatial working memory

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There is much historical evidence that visualization plays a central role in conceptualization processes of physics and in scientific discoveries. Research on the cognitive processes underlying physics discoveries such as Galileo's laws of motion, Maxwell's laws, Faraday's electromagnetic field theory, or Einstein's theory of relativity, has implicated the extensive use of visual/spatial reasoning in these discoveries (Miller, 1986; Nersessian, 1995; Shepard, 1996). Furthermore, the majority of physics problems involve manipulating spatial representations in the form of graphs, diagrams, or physical models, and in fact, the United States Employment Service includes physics in its list of occupations requiring a high level of spatial ability, that is, the ability to perform spatial transformations of mental images or their parts (Dictionary of Occupational Titles, 1991). Despite this evidence, however, relatively little attention has been devoted to understanding the role of spatial visualization skills in physics problem solving. Although research on expert-novice problem

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solving has indicated an importance of visual/spatial representations in physics (e.g., Larkin, 1982; Chi & Glaser, 1988; Ericsson & Smith, 1991), this research has mostly focused on verbal aspects of problem representations as well as semantic and procedural knowledge for solving physics problems. Thus, the goal of the current research was to examine the as-of-yet lesser-studied role of individual differences in spatial ability in physics problem solving.

Spatial ability tests usually involve judgments about the identity of a pair of stimuli presented at different angles (speeded mental rotation tasks) or more complex spatial visualization tasks that require the subjects to generate, maintain, and coordinate information during spatial transformations. Individual differences in spatial ability have been studied since the 1920s when psychometric research first differentiated spatial ability from general intelligence and from verbal and numerical abilities (Smith, 1964). However, the psychometric approach has not provided any clear interpretation of what the differences in spatial ability mean. Beginning in the 1970s, researchers applied the theories and methods of cognitive psychology to the study of spatial ability. One focus of this research has been to interpret individual differences in spatial abilities in terms of working memory. According to Baddeley and Lieberman's (1980) model, working memory consists of separate processing subsystems for visual/spatial and verbal information. Visual/spatial processes such as generating, manipulating, and interpreting visual/spatial images occur in the visual/spatial subsystem called the visuo-spatial sketchpad, whereas verbal processes such as the subvocal rehearsal of words occurs within the verbal subsystem called the phonological loop. Both visual/spatial and verbal subsystems have been shown to have limited processing capacities, and research suggests that spatial ability tests, at least in part, measure differences in visual-spatial working memory capacity. That is, people who differ in spatial abilities also differ in performance on laboratory spatial imagery tasks such as mental rotation (e.g., Carpenter et al., 1999) and measures of spatial working memory (e.g., Salthouse et al., 1990; Shah & Miyake, 1996). Therefore, researchers have proposed that measures of spatial ability tests reflect simultaneous processing and storage demands required to maintain and transform spatial images within the limits of visual-spatial working memory resources (Salthouse et al., 1990; Shah & Miyake, 1996; Miyake, Friedman, Rettinger, Shah, & Hegarty, 2001).

One of the limitations of the above studies, however, was that they ignored "the external validity of the processes that are measured, making it difficult to access the importance of the obtained individual differences in terms of real-world tasks" (Carpenter & Just, 1986, p. 232). Thus, in the current study we focused on examining the relationship between performance on spatial ability tests and solving problems in physics. The domain of physics chosen for this research was kinematics, which describes the motion of objects in terms of such concepts as position, velocity, and acceleration. We chose kinematics because of the diversity of the external visual/spatial representations used in these problems, including graphical schematic representations (vectors of force or velocity; and graphs of motion) and more concrete iconic representations (e.g., of blocks, pulleys, or springs). Additionally, people develop a set of naïve motion principles as a tool for coping with moving objects in everyday life (e.g., Clement, 1983; McCloskey, 1983; Kozhevnikov & Hegarty, 2001), so it is possible to examine their understanding of kinematics, whether correct or incorrect, using qualitative problems that do not require formal knowledge of physics.

Despite the large number of studies on students' difficulties in solving kinematics problems (e.g., Clement, 1983; McCloskey, 1983), very few attempts have been made to relate students' susceptibility to the errors in kinematics problem solving to their spatial ability. However, there is evidence that competing visual/spatial processing demands are more likely to arise as to-be-described kinematics events become more complex. For instance, Isaak and Just (1995) found that participants' susceptibility to incorrect judgments about rolling motion was related to their spatial visualization ability and proposed that the simultaneous processing of the rotation and translation components of the motion overloaded available visual-spatial working memory. Furthermore, Hegarty and Sims (1994) found that the ability to infer how different components of a mechanical system move was related to spatial ability, and Hegarty and Kozhevnikov (1999) showed that this was particularly true when the motion was complex (e.g., a component's motion was constrained by the motions of several other components). Kozhevnikov & Thornton (2006) found that spatial ability was also related to the ability to interpret motion characteristics of an object from a kinematics graph.

To investigate the relationship between physics problem solving and spatial ability, Kozhevnikov, Hegarty, and Mayer (2002a) administered to students a series of different types of kinematics problems along with a number of spatial ability tests. Factor analysis revealed that various types of kinematics problems loaded on different factors. Two particular types of kinematics problems loaded on the same factor as the spatial ability tests. These were problems requiring the extrapolation of an object's trajectory when the correct extrapolation involved combining two vectors of motion, and problems requiring inferences about the characteristics of an object's motion from a different frame of reference. In contrast, other types of kinematics problems, namely one-dimensional motion problems and problems that required the evaluation and calculation of the speed of a single moving object, loaded on a different factor that was interpreted as representing general knowledge of physics laws. Thus, it appears that in addition to conceptual knowledge, spatial ability is particularly related to solving kinematics problems requiring predicting the two-dimensional motion of an object and translating from one frame of reference to another.

The current study focuses on investigating the relationship between performance on spatial ability tests and performance on the above types of kinematics problems in more detail. In Study 1, we investigated the quantitative relationship between students' spatial abilities and patterns of correct and erroneous solutions to different types of kinematics problems. Study 2 further specified the differences in performance between individuals of high and low spatial ability in kinematics problem solving by using a protocol analysis methodology. Finally, in Study 3, we examined the relationship between students' spatial ability and patterns of eye fixations while solving kinematics problems.

## 1. Study 1

### *1.1. Participants*

The participants were 60 undergraduate psychology students, who had not taken any physics courses in high school or college. They were recruited from the Psychology Subject Pool at the University of California, Santa Barbara.

### *1.2. Materials*

The materials consisted of a pre-test questionnaire, eight paper-and-pencil tests measuring spatial ability and verbal ability, and a pencil-and-paper, multiple-choice kinematics problem-solving test.

#### *1.2.1. Pre-test questionnaire*

The pre-test questionnaire included questions about students' high school and college science course work, age, and gender, and Scholastic Aptitude Test (SAT) Quantitative scores.

#### *1.2.2. Kinematics problem solving test*

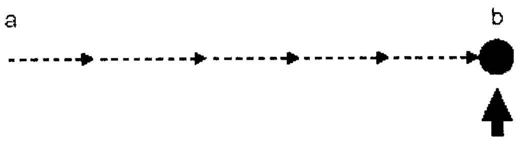
The problems in the Kinematics Problem Solving Test were taken from mechanics diagnostic tests that were designed to assess students' understanding of the laws of mechanics (e.g., Mechanics Diagnostic Test, Halloun & Hestenes, 1985, and the Force Concept Inventory Test, Hestenes, Wells, & Swackhamer, 1992). Three different types of kinematics of problems were included in the test: (1) Two *extrapolation problems* that involved predicting the motion of an object from an observed path—the hockey puck problem (Fig. 1a) and the rocket problem (Fig. 1b); (2) two *graph problems*, one that involved interpreting the movement depicted in a kinematics graph (Fig. 2a) and one that involved relating a graph of one motion component to a graph of another motion component (Fig. 2b); and (3) one *frame of reference problem* that involved the translation from one system of reference to another (Fig. 3).

#### *1.2.3. Spatial ability tests*

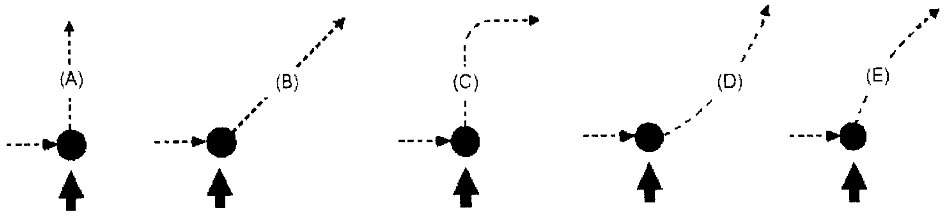
Students' spatial visualization abilities were assessed by the *Paper Folding Test* and the *Form Board Test* (Ekstrom, French, & Harman, 1976). Spatial visualization tests measure processes of apprehending, encoding, and mentally manipulating spatial forms (Lohman, 1988). The Paper Folding Test consisted of 10 items with each item showing successive drawings of two or three folds made to a square sheet of paper. The final drawing in the sequence showed the folded paper with a hole punched through it. The students were to select from among 5 drawings the one showing how the punched sheet would look when fully opened. The Form Board Test consisted of 24 items. Each item showed 5 shaded drawings of pieces of cut foam board, and the cut pieces could be put together to form an adjacently shown target figure composed of all or some of the cut pieces. The students were to indicate which of the pieces, when fitted together, would form the target figure.

Students were also administered two speeded mental rotation tests—the Card Rotation Test and the Cube Comparison Test (Ekstrom, French, & Harman, 1976)—which measure the ability to rapidly judge whether two stimuli show the same figure rotated to different positions in space (Ekstrom et al., 1976; Lohman, 1988). The Card Rotation Test consisted of 10 items with each item requiring participants to view a two-dimensional target figure and judge which of the five alternative test figures were planar rotations of the target figure (as opposed to mirror images) as quickly and as accurately as possible. The Cube Comparison Test consisted of 21 items with each item having drawings of two cubes with letters and numbers printed on their sides. The students were to judge whether the two drawings could be of the same cube.

(a) In the diagram below, you are looking down at a hockey puck sliding at constant speed on a frozen lake from point *a* to point *b*. When the puck reaches point *b*, it receives a single kick in the direction of the heavy print arrow. Assume that the surface of the ice is frictionless.



Which of the path below will the ball follow on the horizontal surface after it receives the kick at b?



(b) The figure shows a rocket coasting in space from *a* to *b* in the direction of the dotted line. Between *a* and *b*, no outside forces act on the rocket. When it reaches point *b*, the rocket fires its engines at a right angle to line *ab* and it keeps firing with a constant force for a certain amount of time T. At this time the rocket will reach point *c* in space.

Which of the trajectories below will the rocket follow from *b* to *c*?

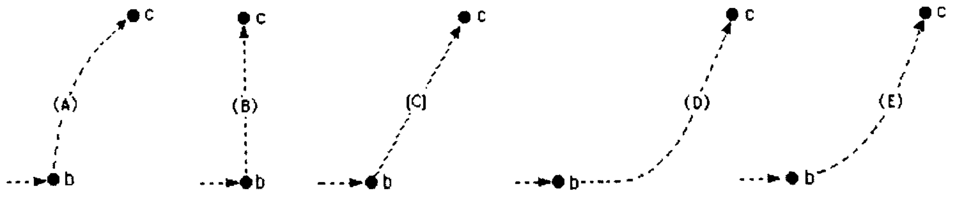
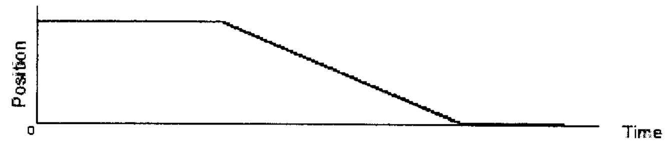


Fig. 1. Examples of extrapolation problems: (a) hockey puck problem; and (b) rocket problem.

### 1.2.4. Verbal ability test

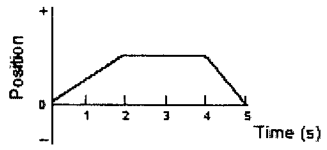
To discriminate between the effects of spatial ability and verbal ability on physics problem solving, students were also administered the *Advanced Vocabulary Test* (Ekstrom, French and Harman, 1976), which measures the "availability and flexibility in the use of multiple

(a) Here is a graph of an object's motion. Which sentence is a correct interpretation?

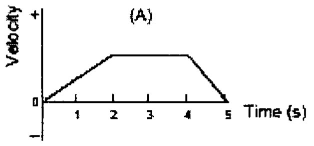


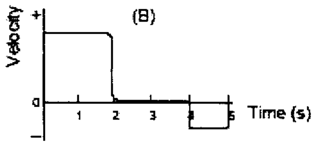
(a) The object rolls along a flat surface. Then it rolls forward down a hill, and then finally stops.
(b) The object doesn't move at first. Then it rolls forward down a hill and finally stops.
(c) The object is moving at a constant velocity. Then it slows down and stops.
(d) The object doesn't move at first. Then it moves at constant speed and then finally stops.
(e) The object moves along a flat area, moves backwards down a hill, and then it keeps moving.

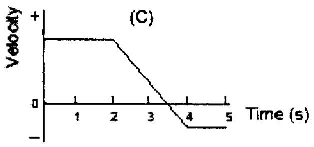
(b) The following graph shows the change in position of an object for 5s time intervals:

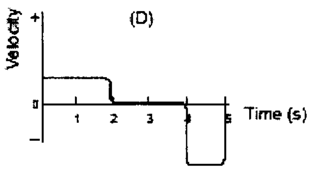


Which of the following graphs of velocity versus time best represents the object's motion during the same time interval?









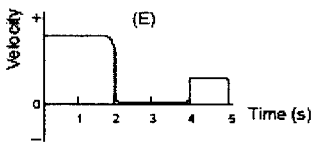


Fig. 2. Examples of graph problems that involved (a) relating a graph to the real-world situation it represents, and (b) relating one type of graph to another.

meanings of words" (Ekstrom, French, & Harman, 1976, p. 163). The test consisted of 18 items. Each item consisted of five numbered words, and the students were to indicate which of these words had the same or nearly the same meaning as a target word that appeared above the numbered words.

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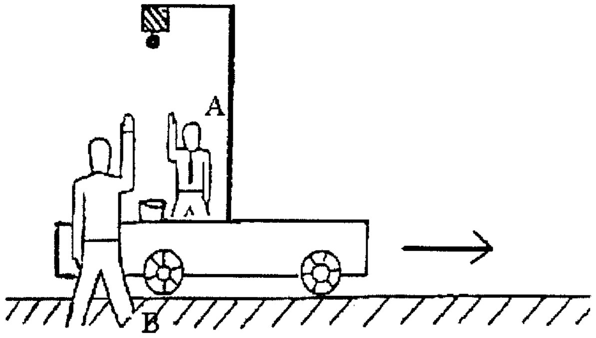

A small metal ball is being held by a magnet attached to a post on a cart. A cup is on the cart directly below the ball. The cart is moving at a constant speed as shown by the arrow in the figure below. Suppose the ball falls off the magnet while the cart is in motion. Observer A stands on the cart, and observer B stands on the road, directly opposite the post of the cart at the moment of ball releasing.

Which of the reports described below corresponds to observer A's view of the falling ball:
(a) The falling ball moves straight down;
(b) The falling ball moves forward;
(c) The falling ball moves backward.

Which of the reports described below corresponds to observer B's view of the falling ball:
(a) The falling ball moves straight down;
(b) The falling ball moves forward;
(c) The falling ball moves backward.

Fig. 3. Example of a frame of reference problem.

### 1.3. Procedure

The tests were administered as part of a larger study, which also included additional types of kinematics problems[1] and mechanical reasoning questionnaires (reported in Kozhevnikov et al., 2002a). The students were tested in small groups of up to six per session. First, they completed the pre-test questionnaire. Then, they were administered the kinematics problem solving test, which they completed at their own pace. Then, students completed the Card Rotation Test, the Cube Comparison Test, and the Paper Folding Test, the Form Board Test and the Advanced Vocabulary Test in that order. Each of these tests was preceded by the standard instructions for that test, and the students were allowed the standard time limits (see Ekstrom et al., 1976) to complete each test.

### 1.4. Results

Descriptive statistics for the spatial and verbal ability tests as well as their internal reliabilities are given in Table 1.

Since all the four spatial tests (i.e., the Paper Folding Test, Form Board Test, Card Rotation Test, and Cube Comparison Test) loaded on the same factor[2] (the factor analysis results were reported in Kozhevnikov et al., 2002a), we created a composite spatial visualization ability

Table 1
Distribution characteristics for spatial ability and vocabulary tests

| Test | Min | Max | Mean | SD | Reliability |
|---|---|---|---|---|---|
| Paper folding | 1.25 | 10 | 6.23 | 2.08 | 0.84 |
| Form board | 2 | 22 | 11.47 | 3.92 | 0.81 |
| Card rotations | 12 | 80 | 58.35 | 16.55 | 0.80 |
| Cube comparison | 0 | 19 | 9.77 | 4.85 | 0.84 |
| Vocabulary test | 1 | 6 | 3.92 | 1.57 | 0.83 |

score for each student by averaging his or her standardized scores ($z$-scores) for each test. Then, the students were classified as having low (the bottom 25% of the distribution with respect to their $z$-score), average (the middle 50%), and high spatial ability scores (the top 25%). Thus, there were 18 low-spatial students and 17 high-spatial students. There were no significant differences between low- and high-spatial groups in either the SAT Quantitative scores, $F(1, 34) = 2.99$, $p = .09$, or Advanced Vocabulary Test scores, $F(1, 34) = 2.54$, $p = 0.12$.

### *1.4.1. Extrapolation problems*

For the puck problem (see Fig. 1a), students read through the problem description and then attempted to choose the correct path, from the five drawn paths, that the puck would follow after receiving the kick. The correct answer to the puck problem is *B*. The puck acquires a constant velocity in the direction of a kick, but the puck's velocity in the initial direction does not change. Therefore, the combination of the initial velocity and the velocity acquired as a result of the kick produces a path at an angle to the initial direction. Similarly, for the rocket problem (Fig. 1b), students read through the description of the problem and then attempted to choose the correct trajectory from among the five trajectories presented below the figure. The correct answer to the rocket problem is *E*. The rocket undergoes a constant vertical acceleration due to the force of the thrust and thus acquires a constantly increasing vertical velocity. Since the rocket's initial horizontal velocity does not change, the combination of the constant horizontal velocity and the continually increasing vertical velocity produces a parabolic arc.

Fig. 4a and 4b show the frequencies of responses chosen by high- and low-spatial ability students for the hockey puck and rocket problems, respectively.

For the hockey puck problem, most of the students (91.4%) chose either response A, B, or E, so we focused our analyses on those responses. As shown in Fig. 4a, the majority of the low-spatial students chose response A, in which they neglected the vertical motion component, while the majority of high-spatial students chose the correct response, B. A large proportion of high-spatial students chose also response E, suggesting that they took into account both motion components but integrated them incorrectly. A test of the likelihood ratio revealed that the relationship between students' responses on the hockey puck problem and their spatial ability was statistically significant, $L^2(df = 2, N = 32) = 9.07$, $p < 0.05$.

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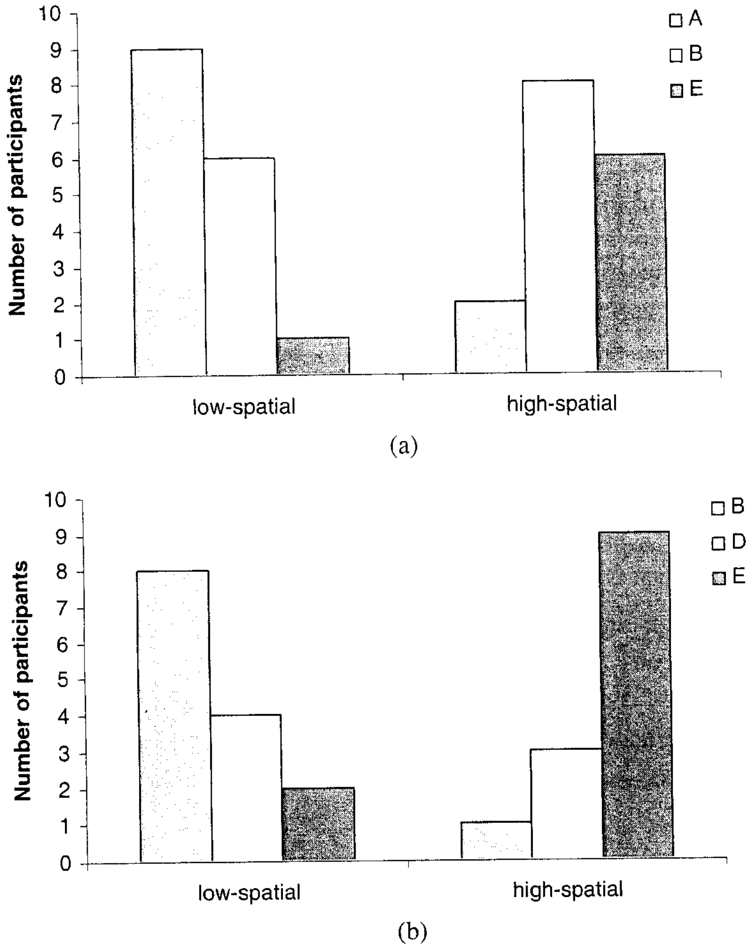

Fig. 4. Students' performance on extrapolation problems with respect to their spatial ability level: (a) hockey puck problem, and (b) rocket problem.

For the rocket problem, most of the students (77.8%) chose the responses B, D, or E. Fig. 4b shows the number of low- and high-spatial students who chose one of these answers. The majority of low-spatial students chose response B, in which they neglected one of the motion components, similar to the hockey puck problem response pattern. In this case, they also failed to take into account the horizontal motion component of the rocket. The majority of high-spatial students, however, chose response E, suggesting that they took into account and correctly integrated both the horizontal and vertical motion components. The relationship between students' types of responses on the rocket problem and their spatial ability was significant, $L^2$ ($df = 2$, $N = 27$) = 11.12, $p < 0.05$.

### *1.4.2. Graph problems*

The two graph problems required students to interpret kinematics graphs that described changes in an object's position over time. In the first problem (Fig. 2a), the graph depicted the motion of the object that at first does not move, then moves at a constant velocity, and later comes to a stop, so the correct answer is D. In the second problem (Fig. 2b), students

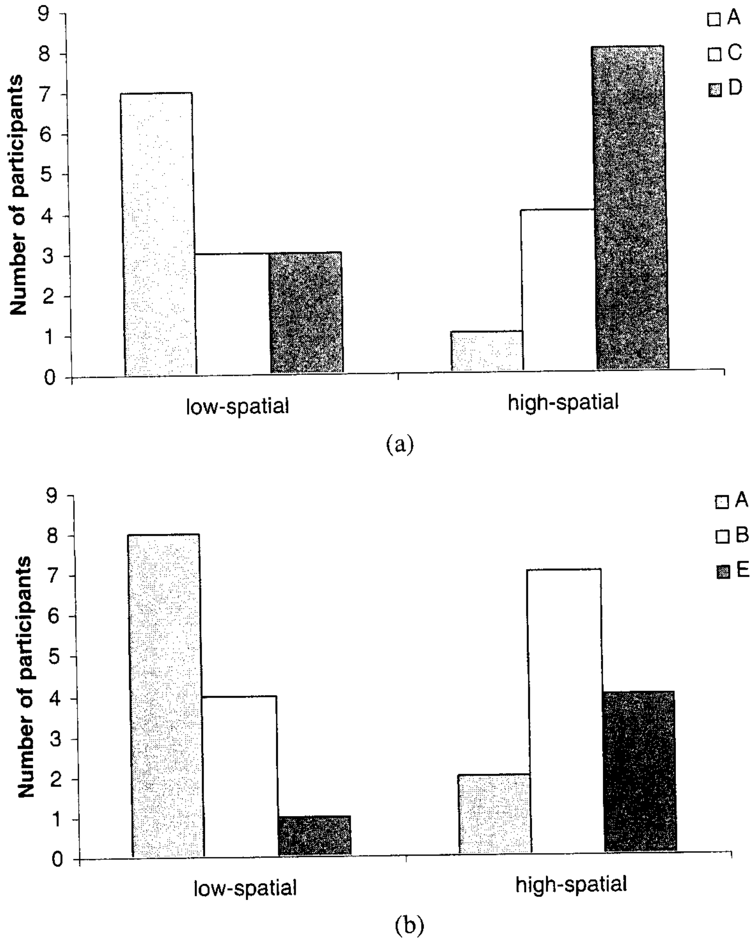

Fig. 5. Students' performance on graph problems with respect to their spatial ability level: (a) for a graph problem that involved relating a graph to the real-world situation, and (b) for a graph problem in which the students were asked to relate one type of graph to another.

were asked to choose from among the five velocity-versus-time graphs the one that correctly represent the objects' motion depicted in the position-versus-time graph. The position-versus-time graph shows that the object moved with constant velocity during the first interval (from 0 to 2), did not move during the second interval (from 2 to 4), and then moved at a constant velocity in the opposite direction and twice as fast as in the first interval (from 4 to 5) before coming to a stop. Thus, the correct velocity versus time graph is B.

Figs. 5a and 5b show the number of the most common responses chosen by high- and low-spatial students for each of the graph problems.

For the first graph problem, most of the students (74.2%) chose response A, C, or D. As shown in Fig. 5a, the majority of the low-spatial students chose response A, giving a "graph-as-picture" interpretation of the object's motion. This kind of literal graph misinterpretation, in which students expect graphs to be a picture of the phenomenon regardless of what the graph ordinate shows (i.e., position, velocity, or acceleration), is a common misconception (McDermott et al., 1987; Mokros & Tinker, 1987; Beichner, 1994). Our results suggest that, when students do not have any prior knowledge about motion graphs, their susceptibility to the graph-as-picture misinterpretations is related to their spatial ability (see also Kozhevnikov, 1999; Kozhevnikov, Hegarty, & Mayer, 2002b; Kozhevnikov & Thornton, 2006). In contrast to low-spatial students, only one high-spatial student chose response A. The majority of the high-spatial students chose the correct response, response D. Interestingly, some of the low- and high-spatial students chose response C, which could be considered an intermediate response between the graph-as-picture misinterpretation and the correct response. These students did not perceive a graph as a holistic picture, but they instead broke it down based on intervals with different characteristics of motion (e.g., a constant velocity at the first interval, slowing down motion at the second); however, their interpretation of the motion on these intervals still had some "pictorial" features (e.g., a flat line represents constant motion independently of what the ordinate shows, etc.). The relationship between students' answers to this problem and their spatial ability was statistically significant, $L^2$ ($df = 2$, $N = 26$) $= 7.56$, $p < .05$.

For the second graph problem, most of the students (74.3%) chose response A, B, or E. As shown in Fig. 5b, the majority of the low-spatial students chose response A, suggesting that the graph would not change. This is again an example of the "graph-as-picture" interpretation, in that the belief that kinematics graphs are like photographs of the situation leads students to conclude that the graph's appearance has no reason to change when the variable on the vertical axis is changed (Beichner, 1994). In contrast to the low-spatial students, the majority of the high-spatial students chose the correct response, B, or the incorrect response, E, which although incorrect it is not a "picture-like" graph misinterpretation. Selection of response E suggests that the student referred to the graph as a schematic representation but failed to realize that the object changed direction on the final interval (from 4 to 5). The relationship between students' answers to this graph problem and their spatial ability was statistically significant, $L^2$ ($df = 2$, $N = 26$) $= 6.61$, $p < .05$.

### *1.4.3. Frame of reference problem*

For the frame of reference problem (Fig. 3), students were asked to predict the trajectory of a ball dropped on a moving cart for two observers, one observer (A) who was moving with the cart and another observer (B) who was stationary on the road. After the ball is released, it would have a horizontal velocity equal to that of the cart, and because the horizontal velocity of the ball is the same as the cart (in the absence of friction) and the cup is directly below the ball, the ball will fall into the cup. Consequently, the correct answers were that observer A will perceive the ball falling straight down, and observer B will see it falling forward in a parabolic arc.

First, our results showed that half of the low-spatial and half of the high-spatial students believed that even for observer A, the ball should fall behind the cup. This is consistent with the findings of other studies (e.g., Fischbein, Stavy, & Ma-Haim, 1989; Eckstein & Kozhevnikov, 1997) indicating that students believe that an object, merely carried by another moving object, does not acquire horizontal velocity, and therefore will fall straight down after being released. Only half of the students were able to predict the correct trajectory that observer A would see, and our further analysis focused only on this group of students. As shown in Fig. 6, for the

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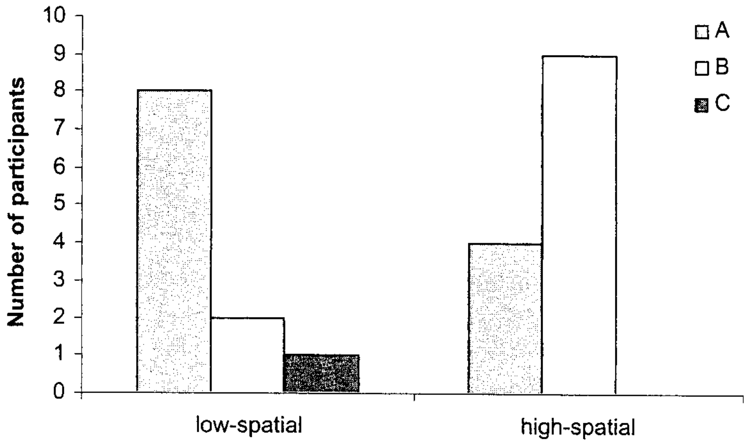

Fig. 6. Students' performance on frame of reference problem with respect to their spatial ability level.

second part of this question, most of the low-spatial students chose response C, which stated that observer B would see the falling ball move straight down.

The students were so strongly influenced by their belief that the ball carried by a cart would fall straight down when released (i.e., would not acquire horizontal velocity) that they failed to correctly infer the motion of the ball across the two frames of reference (i.e., those of observer A and observer B). In contrast, most of the high-spatial students correctly inferred that if the ball was perceived to move straight down into the cup for observer A then it would be perceived to move forward from observer B's point of view in order to hit the cup. There was a significant relationship between students' responses and their spatial ability, $L^2(df = 2, N = 24) = 7.40$, $p < 0.05$.

### *1.5. Discussion*

The results of Study 1 show that a significant correlation exists between students' spatial ability and the overall accuracy of their solutions to kinematics problems, as reported before (Kozhevnikov, 1999; Kozhevnikov et al., 2002a). But beyond this, they demonstrate that there is also a significant quantitative relationship between students' spatial ability and their choice of particular types of solutions to these problems. One possible explanation for this relationship is that the availability of spatial working memory resources directly affects one's ability to solve particular types of physics problems, at least when those types of problems are novel for the student. For instance, in the case of extrapolation problems, a certain amount of visual-spatial working memory might be required to process both the horizontal and vertical motion components and to integrate these components to represent the system's overall motion. Thus, it is possible that the limited visual/spatial resources caused the low-spatial students to neglect one of the motion components. Similarly, interpreting motion graphs requires translating the abstract graphical representation into a real motion event. This translation process, which might place certain demands on visual/spatial working memory, especially when students do not have appropriate knowledge for associating graph features with quantitative relationships. Low spatial students experienced more difficulties than high-spatial students in interpreting graphical information, perhaps because the limited capacity of visual-spatial working memory makes it difficult to break the graph down into intervals with different motion characteristics, and then process the motion at each interval. Furthermore, in the case of frame of reference problems, it makes sense that visual-spatial resources are required to spatially transform one problem representation into another, and the lack of such resources might cause low-spatial students to maintain multiple, isolated views of the same problem.

Alternatively, it is possible that high-spatial students, through their experiences with the external world, have developed more sophisticated ways of analyzing motion phenomena, and this, in turn, advanced the development of their conceptual understanding. For instance, the integration of two motion components as well as the interpretation of graphs as abstract representations exhibited by high-spatial students could be indicative of a more advanced stage of conceptual understanding of motion phenomena rather than a direct reflection of visual/spatial working memory limitations.

## 2. Study 2

Study 2 was conducted to further investigate the differences in solving kinematics problem between high and low spatial ability students as well as to attempt to explore the origin of these differences. In Study 2, students solved the same problems used in Study 1, except that we asked them to "think aloud" while solving open-ended versions of the kinematics problems. Additionally, because hand movements, gestures, and drawings collected concurrently with verbal protocols can provide an effective way to study imagery processes (Clement, 1994), we videotaped students as they solved the problems.

### *2.1. Participants*

The participants were an additional 17 undergraduate students at the University of California, Santa Barbara. These 17 students were selected from a larger group of 49 students on the basis of their scores on tests of spatial visualization and spatial relations (Paper Folding, Form Board, Cube Comparison, and Card Rotation). Eight were high-spatial students (from the top 33.3% of the distribution; 5 males and 3 females) and nine were low-spatial students (from the bottom 33.3% of the distribution; 4 males and 5 females). As in Study 1, to eliminate any possible effects of formal physics training, we chose only students who had not taken physics in college or high school. To control students' quantitative ability, we compared the scores of low-and high-spatial students on the SAT Quantitative test. A one-way ANOVA revealed that there was no significant differences between the two groups on this measure, $F(1, 15) = 1.77$, $p = 0.20$.

### *2.2. Materials*

Students were asked to solve the same problems as in Experiment 1, except that the problems were presented in open-ended format, rather than a multiple-choice format. Each extrapolation problem (hockey-puck and rocket) was presented to the student with a diagram, similar to the diagrams in Fig. 1a and 1b, upon which they were asked to predict, draw, and explain the future motion of an object. For the first graph problem, the students were presented with the graph shown in Fig. 2a and asked to visualize and describe a real situation depicted by the graph. For the second graph problem, students were asked to draw a graph of velocity versus time based on the graph of position versus time shown in Fig. 2b.[3] Finally, for the frame of reference problem (see Fig. 3) students were asked to describe the path of the ball that observer A and observer B would see.

### 2.3. Procedure

The spatial ability tests (Paper Folding, Form Board, Cube Comparison, and Card Rotation) were administered as in Study 1. On another day, the kinematics problems were presented, and students were interviewed about their solution methods. During the interviews, students were asked to solve the same types of kinematics problems used in Study 1. However, the problems were not presented in a multiple-choice format as in Study 1 but in open-ended format. As mentioned above, the students were also videotaped while solving the problems, and their hand movements, gestures, and drawings were assessed in addition to their verbal responses. Two independent raters analyzed the videotapes coding for the types of responses and gestures made (as described below in the following sections). Their agreement regarding students' types of responses was 0.97 and disagreements were resolved by consensus.

### 2.4. Results

#### 2.4.1. Extrapolation problems

*2.4.1.1. Low-spatial students:* Analysis of students' protocols confirmed the results of Study 1 that most low spatial ability students consistently neglected the horizontal motion component while solving the extrapolation problems.

For instance, for the hockey puck problem, seven low-spatial students only took into account the vertical motion component while solving the problem. For example, one student said, "*I think it will go this way... straight up... No force acts on it, so if just kicked, it will go the direction of the kick...*" One of the nine low-spatial students stated that the direction of motion would depend on how strong the kick was. However, as revealed by his answer, the student only considered one component of motion (horizontal or vertical) at a time, "*It depends on how strong the kick will be. If this is a strong kick, then I think the ball will go in the direction of the kick, but if the force that moves the ball was stronger than the force of a kick, and the ball has a high speed, then the ball will continue to move horizontally, like it moves before...*" Thus, this student recognized both components but did not attempt to integrate them. Only one of the nine low-spatial student predicted that "*after the kick, the puck would move at some angle to its initial direction,*" thus showing evidence of considering both components simultaneously.

A similar pattern was revealed for the rocket problem. Seven low-spatial students stated the rocket would go straight up from point *b*. The following were typical answers given by low-spatial students:

> Low-spatial student #1: "*The rocket is going up . . . I don't understand what I am supposed to prove . . . It will just go up. . . . Yes, it just moves up . . .*"
> Low-spatial student #2: "*I say it will just go up . . . Space, no gravity, I guess, there are no other forces . . . It does not matter if it's going faster or not, I believe it does not matter, it will go straight up . . .*"

*2.4.1.2. High-spatial students:* All of the high spatial students, in contrast to low-spatial students, consistently referred to both motion components in the hockey puck and rocket problems, although in several cases they integrated the two motion components incorrectly. Their typical solutions for hockey puck problem are illustrated in the following transcripts:

> High-spatial student #1: "*The puck initially travels this direction . . . Then second force acts upon it. It does not overcome the first force, but interacts with the first force, so that together . . . together they both come here (the student made a hand movement to show integration of forces), they provide this direction . . .*"
> High-spatial student #2: "*The first force is the initial force with which the puck is sliding around . . . The second force is from the kick: when it kicked the puck, it did not immediately come this way (the student showed the vertical direction), nor did it just ignore the kick* (the student showed a horizontal direction). *The puck will go this way* (the student showed a diagonal direction)."

The typical high-spatial students' solutions for the rocket problem are illustrated in the following transcripts:

> High-spatial student #1: "*It follows from a to b without any external forces. But there is a force that somehow was applied to the rocket. Let's say this is an initial force due to which the rocket is moving with a constant speed from a to b. Once the engine starts to work, there is also a force from the engine. There is no difference from the previous case of the puck . . . Oh, there must be a difference . . . There is constant acceleration because the rocket keeps firing: this means that we have constantly increasing acceleration this way. It will be like this* (The student drew a parabolic arc)."
> High-spatial student #2: "*It's going with a constant speed, right? It will be like that* (drew a parabolic arc). *It still got this velocity* (drew a horizontal line). *It's going straight* (showed the horizontal direction), *it never stops unless force acts on it and no force acts on it; and if force acts up it will continue to go up also; but it also continues to go to the right. But if there is absolutely no friction, it would go this way* (the student drew diagonal straight line instead of parabolic arc)."

The relationship between the students' spatial ability and their responses to extrapolation problems (coded as taking into account a single or both motion parameters) was significant for both the hockey puck, $L^2(df = 1, N = 17) = 17.23$, $p < 0.001$, and the rocket problems, $L^2(df = 1, N = 17) = 10.62$, $p < 0.01$. These results are consistent with the results of Study 1: low-spatial students were more likely than high-spatial students to neglect one of the motion components. Even after considering the possibility of the existence of two motion components (e.g., "*It depends on how strong the kick will be*"), the low-spatial students still did not perceive their integration as relevant.

As can be seen from their answers, both low- and high-spatial students exhibited naive misconceptions about motion. They referred to some "driving force" or "initial force" that was applied to the rocket or to the puck in order to keep these objects in motion, in contrast to Newtonian theory, stating that the objects remain in motion at a constant velocity (or rest) unless acted upon by an external force, so no force is needed in order for an object to keep its velocity. Although such misconceptions were present in both groups of students, the responses of high-spatial students were somewhat more sophisticated. For instance, one high-spatial student expressed that "*there is constant acceleration because the rocket keeps firing*", and that "*it does not overcome with the first force, but interacts with the first force*" suggesting that high-spatial students might have developed more sophisticated understanding of the laws of motion in comparison with low-spatial students.

### *2.4.2. Graph problems*

*2.4.2.1. Low-spatial students:* While visualizing the situation depicted on the graph, all nine low-spatial students interpreted the graph literally, as a picture. These are the typical answers of low-spatial students:

> Low-spatial #1: "*Could it be just elevation or height? And then a hill?*"
> Low-spatial #2: "*The car goes constantly and then goes downhill... It does not change its direction. It goes downhill. This is a hill.*"

As shown in the above transcripts, no attempts were made to break the graph down into smaller intervals that had different characteristics of motion or to consider the graph as an abstract representation of an object's motion. Even when asked by the interviewer to visualize the motion of an object at each interval in succession and to describe the changes in the object's velocity from one interval to another, low-spatial students encountered difficulties, as evident form the following transcripts:

> Low-spatial student #3: " *I imagine a ball goes downhill.*"
> Interviewer: "*Could you visualize the motion of an object at the first interval? At the second? How does its velocity change from the first interval to the second?*"
> Low-spatial student #3: *It's moving at a constant speed at first, then it's slowing down and finally comes to a stop, but I can not really imagine this. For example, ball... It is definitely a ball going downhill... or uphill?* (showed on the graph how a ball is moving uphill in the reverse direction from right to the left side of the graph).

Similarly, while solving the second graph problem, the low-spatial students believed that switching the ordinate variable from position to velocity would not change the appearance of the graph. In attempting to compare one type of graph to the other, these students did not ignore the shape of the original graph. The following are the typical answers of low-spatial students for the second graph problem:

> Low-spatial student #1: "*It will be similar to the graph of position; the same thing. Yes, I would say this is the same thing.*"
> Low-spatial student #5: "*This is the same thing as position versus time, isn't it?*"

One of the low-spatial students applied a mathematical strategy to solve this problem, but was confused by the fact that the graph derived by the mathematical formula was different from the original graph: "*Velocity versus time is always the derivative of the position* (calculates mathematically the derivative for each interval and draws the correct graph for velocity vs. time). *Hmm, I am really surprised. I thought I knew calculus. This should be the same graph as position versus time, but mathematically I got a different one.*" Interestingly, the student broke the graph down into intervals to calculate a derivative at each interval, but he made no attempts to restructure his literal graph interpretation and visualize the object's velocity at each interval successively.

2.4.2.2. *High-spatial students:* All of the high-spatial students gave descriptions of the situation that were primarily schematic, as illustrated in the following transcripts:

> High-spatial students #1: "*At the first interval of time the position is the same: it can not move. It has a constant velocity at the second interval. It is moving constantly at a constant speed.*"

None of the high-spatial students referred to the graph as a concrete duplication of the motion event. Instead, they broke the graph down into smaller intervals, and their mental transformations were mostly focused on visualizing how the object's velocity changed from interval to interval. Seven high-spatial students were able to interpret the motion correctly. The other two students incorrectly assumed that at the last interval the object would come back to its original place:

> High-spatial student #6: "*At first, something was not moving, just staying. Then it begins to move. Say it's a car. It was staying some period of time and then, going back with a constant velocity to its original place.*"

For the second problem, while relating one type of graph to another, all of the high-spatial students believed that changing the ordinate variable from position to velocity would change the appearance of the graph. However, only three of them solved the problem by applying a visual strategy. This is an example of one of their answers:

> High-spatial student #7 "*It seems it has pretty constant velocity at the first interval. At the second interval the position is the same, it cannot move. It is constantly moving at the third interval; it is just moving at a constant speed.*"

The other high-spatial students used analytical strategies (however, three of these were not able to derive the correct mathematical solution).

The relationship between students' spatial ability (low versus high) and their responses to the first graph problem (coded as pictorial versus schematic interpretation) was significant, $L^2(df = 1, N = 17) = 23.51$, $p < 0.001$. Similarly, for the second graph problem, there was a significant relationship between spatial ability and the type of response (pictorial, schematic, analytical), $L^2(df = 1, N = 17) = 23.51$, $p < 0.001$.

The above results are consistent with recent findings on the relationship between graph interpretation and spatial ability (Kozhevnikov, Hegarty, & Mayer, 2002b; Kozhevnikov, Kosslyn, & Shephard, 2005). They also suggest that low- and high-spatial students use different strategies to interpret graphs. Low-spatial students tended to interpret graphs by looking for a pattern in their visual memory that matched to the stimulus input (e.g., the shape of a hill). In contrast, high-spatial students tended to interpret graphs as an abstract spatial configuration, by breaking the graph down into intervals and visualizing changes in an object's velocity from one interval to another.

### 2.4.3. *Frame of reference problem*

*2.4.3.1. Low-spatial students:* Consistent with the naïve belief described in Study 1 that an object carried by another moving object does not acquire horizontal velocity and therefore will fall straight down after being released, four low-spatial students stated that the ball would fall behind the cup and that both observers would see it falling backward. However, five of the low-spatial students were able to correctly predict the perceived path of the ball in observer A's moving frame of reference. In spite of this, however, four of them continued to believe that observer B would see the ball fall straight down and miss the cup. These are typical answers of these low-spatial students:

> Low-spatial student #1: "*If I move with a cart, the ball will fall straight down into the cup. I am guessing. It will go into the cup because the ball will fall straight down, because there is no air resistance. I think both observers A and B will see the ball falling straight down. Both will see the same straight path for the ball.*"
> Interviewer: "*But if B sees a ball fall straight down, the ball will miss the cup.*" Low-spatial student #4: "*Why should both observers see the ball falling into the cup? These two things are independent.*"
> Low-spatial student #5: "*If I am not in the cart, it does not move with the cart. The ball definitely will miss the cup; the ball does not move with the vehicle, it's just going down. If I am moving with a cart, it will probably . . . let's see . . . I don't know it would probably goes into the cup. I will see it falling straight down. I will see it going down into the cup. It's perception . . . Right? Some kind of optical illusion?*"
> Low-spatial student #3: "*If I stay in the car and drop a penny, it always falls straight down. The ball will fall into the cup. But for the viewer B it should fall backward, because once the ball is released . . . the car is moving forward and the ball is falling backward because it's losing velocity. Hmm. It's contradictory to what I said before . . . It's losing speed due to gravity. I would think it will fall into the cup, but taking into consideration these circumstances. I think observer B will see the ball missing the cup and falling behind it.*"

*2.4.3.2. High spatial students:* Three high-spatial students reported that the ball would fall behind the cup and that both observers would see it falling backward. The other five high-spatial students correctly stated that they would see the ball falling straight down into the cup from observer A's frame of reference, and this inference then helped them to imagine the correct perceived movement of the ball from observer B's frame of reference. In contrast to the low-spatial students, all five of these high-spatial students were able to dynamically transform the representation of the problem from observer A's moving frame of reference to observer B's stationary frame of reference. The following are typical answers of these high-spatial students:

> High-spatial student #1: "*If I stand on the car, I will see it falling right in the cup. If I stand on the road, I will see the ball definitely should fall forward to fall into the cup (showed two lines with hands: one for the moving cup and another for the ball falling forward; at some point the two hand movements met each other). The ball will fall right in the cup, because it's moving with a cart. It is moving with the cart when it's released.*"
> High-spatial student #2: "*No wind or friction? For me in the cart, if I am looking down I will see the ball falling straight down into the cup. To the viewer B then, it must go forward* (showed by hand a parabolic arc). *It's not exactly what I thought before. Hmm. It means that for observer B the ball*

*should fall straight down theoretically, but it won't, probably, because both the ball and cart are going at the same speed. The ball is still going in the direction of the cart while falling.*"

For those students, who were able to correctly predict the trajectory of the ball for observer A, their responses were coded as those in which observers A's and observer B's views of the falling ball were perceived to be coordinated and those in which they were not. The results of the likelihood ratio test indicated that the relationship between students' spatial ability and their response to the frame of reference problem (coded as whether the correct trajectory was given) was significant, $L^2(df = 1, N = 10) = 8.46$, $p < 0.01$, confirming the findings of Study 1 that low-spatial students are more likely to view the same problem as two different ones when asked to imagine the motion from different frames of reference. Their multiple-view representations of the same problems were isolated from, and often at odds with, each other.

Both low- and high-spatial students who were able to correctly predict the trajectory of the ball for observer A exhibited the same misconception (i.e., they believed that an object carried by another object does not acquire horizontal velocity). However, high-spatial students noticed the conflict between two representations, and changed their reasoning accordingly. This is evident from one the high-spatial student's responses: "*It means that for observer B the ball should fall straight down theoretically, but it won't, probably because both the ball and cart are going at the same speed. The ball is still going in the direction of the cart while falling.*" In contrast, low-spatial students did not see the conflict in aligning the two spatial representations, and thus were unable to resolve the conflict.

### 2.5. Discussion

Overall, the findings of Study 2 are consistent with those of Study 1. In contrast to the high-spatial students, low-spatial students did not combine the two motion components, tended to interpret graphs as pictures, and did not coordinate two frames of reference. Findings from the extrapolation problems showed that low-spatial subjects consistently neglected the relevance of one of the motion components, suggesting that they tended to choose those solutions that tax visual-spatial working memory less. Based on the more advanced interpretations of the extrapolation and graphs problems by high-spatial students, one might argue that the tendency of high-spatial students to take both motion components into account as well as to interpret graphs schematically is a result of deeper conceptual understanding of physics phenomena rather than visual-spatial processing resources. In the case of the frame of reference problem, however, both low- and high-spatial students started with the same impetus misconception about motion (i.e., that an object carried by another moving object does not acquire horizontal velocity). However, the reports given by high-spatial students who correctly predicted observer B's perception suggest that their ability to coordinate frame of references appeared to be critical in bringing about a conceptual change. Thus, it is likely that both spatial competence and conceptual understanding are at play: some basic abilities in spatial processing (e.g., the ability to integrate two components, to parcel a visual representation into parts; and to perform spatial transformations) can bring about significant changes in students' conceptual understanding, that in turn might influence students' future strategies and responses.

## 3. Experiment 3

To further investigate how spatial ability affects the solution of kinematics graph problems and two-dimensional motion extrapolation problems, we examined students' eye fixations while solving these problems in Experiment 3. Eye movements have been used to study visual imagery processes (Brandt & Stark, 1997; Spivey & Geng, 2001; Laeng & Teodorescu, 2002) as well as cognitive processes underlying visual/spatial tasks, such as mental rotation (Carpenter & Just, 1986) mechanical reasoning (Hegarty, 1992), and graph comprehension (Carpenter & Shah, 1998). In this study we examined differences in the eye fixations of low-spatial and high-spatial students as they solved motion extrapolation and kinematics graph problems.

Recent eye-movement studies have found that mental imagery is often accompanied by eye movements. For example, when viewing a static scene and imagining motion, people's eye-fixations mimic the direction of imagined motion (Spivey & Geng, 2001). In Study 3 we used this observation to examine differences in visualization processes between high- and low-spatial students in solving motion extrapolation problems. We predicted that if the students use spatial imagery rather than just conceptual physics knowledge to predict the trajectory, they would make eye fixations in the direction of the trajectory of motion.

In the case of graph comprehension, Carpenter and Shah (1998) have proposed that graph comprehension occurs through integrative processing cycles in which learners identify visual patterns in the graph (e.g., positive slopes vs. negative slopes), translate the patterns into quantitative and qualitative interpretations (e.g., interpreting an upwardly curved line as an accelerating function), and then relate the interpretations to the referents inferred from the labels on the graph. They provided evidence for this process by examining participants' eye-fixations on graphs during comprehension. A striking result of their study is that participants spent most of their time fixating the axes and labels of the graphs, and that graph comprehension involved frequent switches of gaze between the graph patterns, axes and labels, presumably to integrate the information in the graph patterns with that presented in the axes and labels. We expected high spatial individuals to interpret the motion graphs in this way, showing frequent fixations and a high proportion of time on the axes and labels of the graph. This should be particularly true if they break the graph down into intervals showing different motion events. On the other hand, because low-spatial individuals appear to interpret graphs as pictures and view them holistically rather than interval by interval, we predicted that they should spend less time and have fewer fixations on the axes and labels.

We did not investigate the frame-of-reference problem in Study 3, because this problem required the use of more complex, dynamic visual/spatial imagery (e.g., imaging the cart moving, the ball being released, and the perspective changes), and we could not make predictions about the eye-movement patterns that would accompany such complex visual/spatial imagery processes.

### *3.1. Method*

#### *3.1.1. Participants*

Fifteen students were recruited from the general participant pool in the Psychology Department at Rutgers-Newark. The participants were selected based on their Paper Folding test scores, and the Paper Folding Test was administered under a variety of contexts for studies to be reported elsewhere. Six low-spatial ($M$ paper folding = 1.99) and nine high-spatial ($M$ paper folding = 8.22) students agreed to participate in the eye-tracking study.

### *3.1.2. Materials*

There were two kinematics problems used in this study. The first problem was the hockey puck problem in Fig. 1a, which was not accompanied by any text other than the labels for points $a$ and $b$. When Fig. 1a was presented, the experimenter read the description and asked the student to imagine and describe in as much detail as possible the path that the puck was likely to travel after receiving the kick. The second problem showed a position-versus-time kinematics graph, similar to the graph in Fig. 2a. It was not accompanied by any text other than the labels for the axes, and the experimenter stated only that it was "a graph of an object's motion" and that the student was to study the graph and to describe in as much detail as possible what was depicted.

The graph and figure were presented on a 17-inch Dell monitor. The students' eye-movements were measured via an iView-X RED eye tracking system designed by Senso-Motoric Instruments, Inc. (Boston, MA), and their descriptions were recorded on audiotape. A chin-cup and headrest were used to keep the students' viewing distances constant and to reduce the students' head movements, although the eye-tracking system was set to accommodate slight head movements. Finally, the eye-tracking system was calibrated for each student at the beginning of the session using the 9-point iView-X calibration system, and, if necessary, the eye-tracking system was adjusted for drift between the presentations of the problems.

### *3.1.3. Procedure*

The students were tested individually. They were told that they would be shown some figures and would be asked to answer some questions about the figures while their eye-movements and answers to the questions were recorded. Prior to showing each figure, the student was asked to fixate on a set of cross-hairs. After the student fixated on the cross-hairs, the experimenter gave the student a verbal "get ready" prompt and started recording the student's eye-movements. The experimenter then pressed a key on the keyboard to display the figure, and after it appeared, the experimenter read the problem to the student. If asked, the experimenter re-read the problem.

The position-versus-time graph problem was presented first, followed by the motion extrapolation problem. For each problem, the student was given unlimited time to respond. When a student's description was ambiguous, the experimenter asked the student to elaborate, and at the end of the session, the experimenter had the students sketch the trajectory for the motion extrapolation problem.

## *3.2. Results*

### *3.2.1. Extrapolation problem*

None of the low-spatial students accurately described the puck's trajectory; however, seven of the nine high-spatial students did. A likelihood ratio revealed that the association between

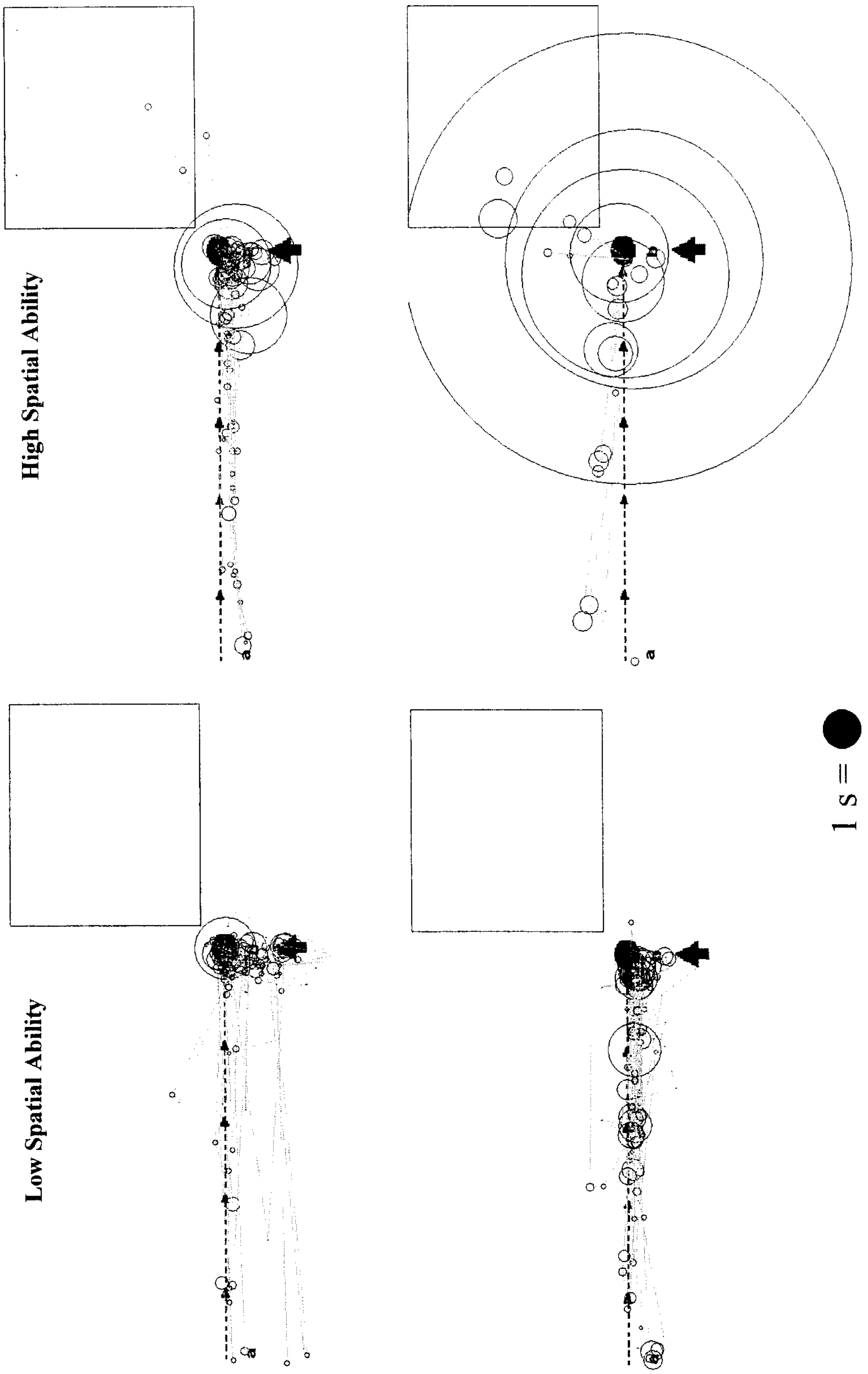

Fig. 7. The fixations and scan paths of two low-spatial and two high-spatial ability students while solving the hockey puck problem. Circles represent fixations; the diameter of the circle represents the duration of the fixation. Lines represent the scan paths. Rectangles mark regions of interest (ROI) that we defined.

spatial ability and correctly describing the puck's trajectory was significant, $L^2$ ($df = 1$, $N = 15$) $= 11.19$, $p < 0.01$, $\Phi = 0.76$. The two high-spatial students who incorrectly described the trajectory failed to integrate the horizontal motion component (i.e., they predicted that the puck would move only in the direction opposite to the kick), and the data for these students were not further analyzed.

If students are solving this problem by visualization (imagery) strategies, we might expect them to make eye fixations in the direction that the puck would travel (Spivey & Geng, 2001). Thus, we examined whether a student made eye movements and fixations in the correct direction. This was considered to have occurred if the gaze path was in the direction of motion and the fixation fell within a region of interest (ROI) defined as a rectangular space beginning 1 cm to the right and 1 cm above point *b* and extending to the top and right edges of the display (see Fig. 7). Fixations were defined as periods in which the eye remained within a circular area (diameter = 19.5 cm) for at least 50 ms.

Five of the seven high-spatial students who answered this problem correctly made at least one fixation within the ROI, whereas only one of the low-spatial students made a fixation within the ROI. A likelihood ratio revealed that the association between spatial ability and fixating within the ROI was significant, $L^2$ ($df = 1$, $N = 13$) $= 4.16$, $p < 0.05$, $\Phi = 0.55$. Thus, the high-spatial students who correctly described the trajectory showed evidence, in the form of eye movements and fixations, of imaging the path that the puck would travel. One of the other high-spatial subjects who answered correctly did not make any eye movements away from the elements shown in the figure. The seventh high-spatial subject made upward and rightward eye movements and fixations from point *b*, suggesting that he was actually visualizing the results of the two motion components.

Three students who reported that the puck would move opposite to the direction of the kick did make at least one fixation in that direction, but none made eye-movements and fixations in the direction of the horizontal motion component, thus failing to show evidence of visualizing this horizontal component.

### *3.2.2. Kinematics graph*

None of the low-spatial students gave correct schematic descriptions of the graph. Five of the six low-spatial students gave "graph-as-picture" descriptions, and one failed to describe the graph in terms of an object's motion. Six of the high-spatial students, however, gave correct schematic descriptions of the graph. Two gave "graph-as-picture" descriptions, and one incorrectly described the graph in terms of an accelerating object. Because our primary interest was in the more typical low-spatial students who gave "graph-as-picture" descriptions and high-spatial students who gave correct schematic descriptions, and because the number of low- and high-spatial students giving other types of responses was small, only the data for the low-spatial students who gave "graph-as-picture" descriptions and the high-spatial students who gave correct schematic descriptions were further analyzed.

For the position versus time graph, regions of interest (ROI) were defined in an attempt to isolate fixations on the x- and y-axes, the labels for the axes, and the line segments within the graph (see Fig. 8) and then to calculate the proportion of fixations made on the axes, labels for the axes, and the line segments out of the total number of fixations made and the proportion of time spent studying these features out of the total duration spent studying and describing

Table 2
Mean proportion of fixations and proportion time spent studying the regions of interest on the graph by low-spatial students who gave pictorial descriptions and high-spatial students who gave correct abstract descriptions

| | | Proportion of Fixations | | Proportion of Time | |
|---|---|---|---|---|---|
| Region of Interest | Spatial Ability | *M* | *SE* | *M* | *SE* |
| Line segments | Low | 0.55 | 0.09 | 0.48 | 0.10 |
| | High | 0.44 | 0.04 | 0.38 | 0.04 |
| Axes | Low | 0.20 | 0.04 | 0.16 | 0.02 |
| | High | 0.31 | 0.05 | 0.29 | 0.04 |
| Labels | Low | 0.07 | 0.03 | 0.07 | 0.03 |
| | High | 0.10 | 0.02 | 0.09 | 0.02 |

the graph. Fixations were defined as periods in which the eye remained within a circular area (diameter = 19.5 cm) for at least 50 ms.

Separate between-groups ANOVAs were calculated for the proportion of fixations within and proportion of time spent studying each graph element (axes vs. labels vs. line segments) for the low and high spatial ability groups (see Table 2, and for examples, see Fig. 8). The analyses revealed that the high-spatial students spent a greater proportion of time studying the axes than the low-spatial students, $F(1, 9) = 7.00$, $p < 0.05$, *partial* $\eta^2 = 0.437$, and they also tended to make a greater proportion of fixations along the axes, $F(1, 9) = 3.11$, $p = 0.11$, *partial* $\eta^2 = 0.258$. The groups, however, did not significantly differ in the proportion of time spent studying or the proportion of fixations made along the line segments or axis labels, all $ps > 0.22$. Due to some potential uncertainty regarding the focus of some of the fixations along the line segment and x-axis segment on the right of the graph, we calculated the proportion of time studying and proportion of fixations made along just the horizontal line segment on the right and slanted line segment, but the differences between the low and high-spatial groups were still not significant, both $Fs(1, 9) < 1$.

Thus, the data showed that high-spatial students who accurately described the kinematics graph studied the graph differently from low-spatial students who gave "graph-as-picture" interpretations of the graph. In terms of Carpenter and Shah's (1998) model of graph comprehension, the fixation data and descriptions from the present study show that high- and low-spatial students spent equal proportions of time studying the line segments, suggesting that both at least engaged in pattern encoding. High- and low-spatial students also spent similar proportions of time studying the axis labels, which suggests that both considered the importance of interpreting the patterns according to the variables. However, the finding that the high-spatial students studied the axes more than the low-spatial students and that they gave interval-by-interval descriptions of the functional relations depicted in the graph (e.g., *In the first couple of seconds, the object it is staying still, and then for like 2 seconds it's moving, the position is changing, and then it's still again*) suggests that the high-spatial students engaged in the second and third processes of Carpenter and Shah's (1998) graph comprehension model (i.e., translation the visual pattern into conceptual relations and by identification of the referents of the relations). That is, they relied more extensively than low-spatial students on

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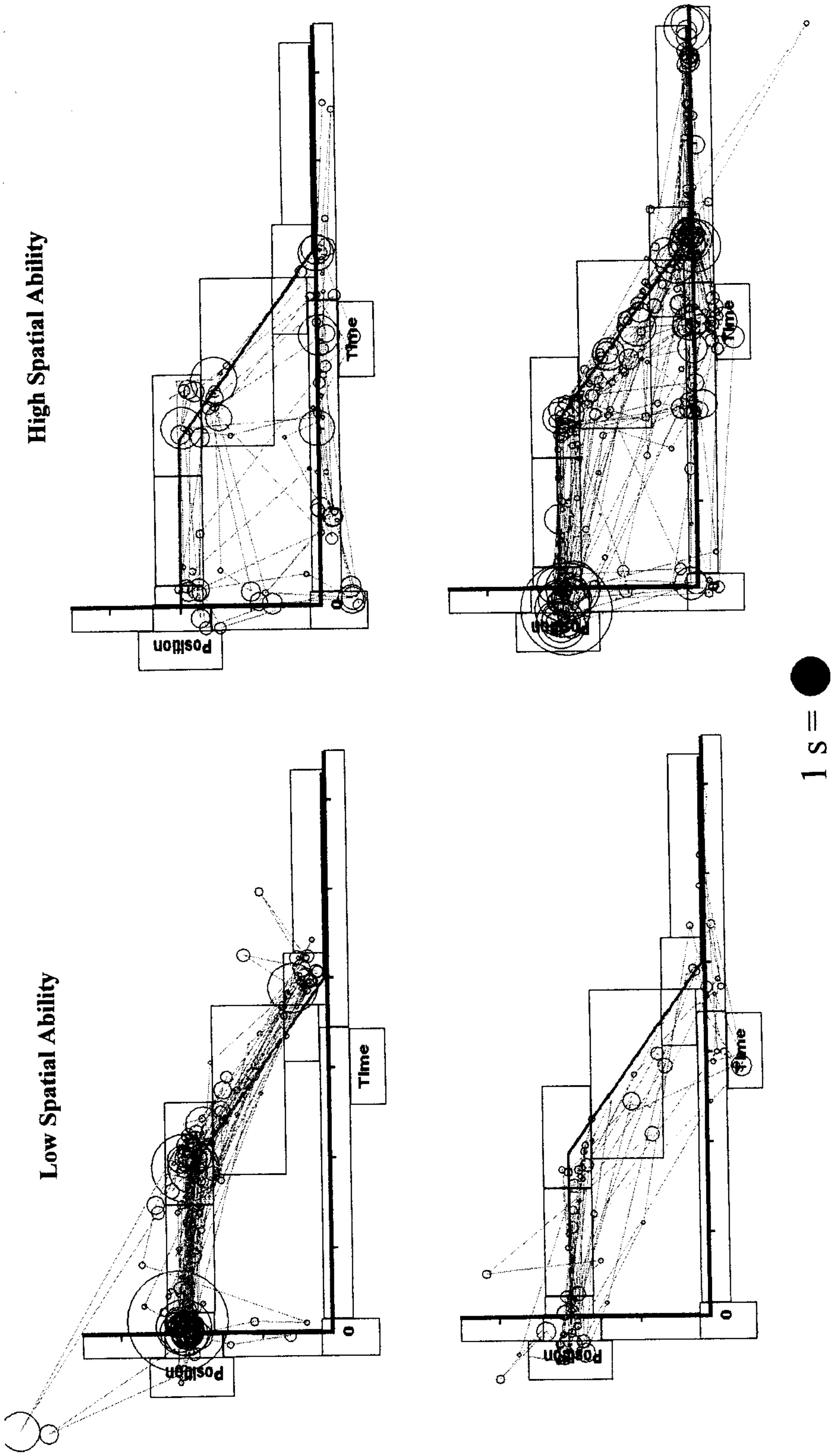

Fig. 8. The fixations and scan paths of two low-spatial and two high-spatial ability students while studying a kinematics graph. Circles represent fixations; the diameter of the circle represents the relative duration of the fixation. Lines represent the scan paths. Rectangles mark areas of interest that we defined.

the scales when deriving the functional relations depicted in the graph and when associating those relations to the concept of an object's position changing over time. Consistent with the integrative processing proposed by Carpenter and Shah, high-spatial students tended to study the axes, axis labels, and line segments repeatedly.

The low-spatial students who gave incorrect descriptions, on the other hand, relied less on the scale information available on the axes, and their descriptions show that they failed to translate the patterns into integrated functional relationships. Instead, they appeared to have retrieved memories of a motion event that matched the general shape of the line segments (e.g., *Well, someone could be diving off a diving board into a pool and then swimming to the end of the pool*), giving pictorial responses consistent with the patterns of the low-spatial students in Studies 1 and 2. The difference between the high- and low-spatial students in translating the patterns into integrated functional relationships suggests that spatial resources are important for such translations.

## 4. General Discussion

The current research provides insights into the nature of individual differences in spatial ability as well as the types of physics problems that might require high visual-spatial processing resources. Study 1 showed that there was a significant relationship between visual-spatial abilities and the solutions that students gave to kinematics problems. In particular, high-spatial students were more likely to take into account and successfully integrate several motion parameters, to interpret kinematics graphs as abstract representations of an object's motion, and to reorganize one spatial problem representation into another coordinated, corresponding representation. In contrast, the low-spatial students were more likely to consider a single motion parameter at a time, to interpret graphs as picture-like representations, and to hold multiple, uncoordinated representations of the same problem. The results of Study 2 were consistent with the findings from Study 1 in that students' solutions to the kinematics problems presented in these studies were related to spatial visualization ability. In particular, the students' transcripts showed that low-spatial students did not integrate the two motion components in the case of the extrapolation problems, did not decompose graphs into intervals with different motion characteristics, and did not spatially transform one problem representation to another.

Finally, the results from Study 3 provided further insight into the influence of visual/spatial ability on solving kinematics problems. First, the fact that most of the high-spatial students made upward and rightward eye movements as well as eye movements in the diagonal direction that puck would travel suggest that they were actually visualizing the results of integrating two motion components. In contrast, most of the low spatial students did not make eye movements in the direction of the horizontal motion component showing no evidence of visualizing the horizontal component. Second, consistent with Studies 1 and 2, the availability of spatial resources predicted how the low- and high-spatial students studied and interpreted the graphs. High-spatial students who interpreted the graphs as a schematic representation of an object's motion spent a greater proportion of time studying the axes of the graph than low-spatial students who interpreted the graph as a literal representation of an object's motion, and the high-spatial students were able to integrate both the position change and the time dimensions.

The data reported in this paper are correlational in nature, so we cannot rule out the possibility that some other variable covaries with spatial ability in our samples and is causing the differences in kinematics problem solving that we observed. Some plausible correlates that might cause differences in performance include general intelligence, mathematical ability, and conceptual knowledge. It is unlikely that the difference reported in our studies were due to the differences in intelligence or mathematical ability. We measured verbal intelligence in Study 1 and found no significant difference on this measure between our high- and low-spatial groups. Furthermore students in all of our studies reported their SAT Quantitative scores and again there was no significant difference between high- and low-spatial students on this measure.

To control for possible differences in conceptual knowledge, we recruited as participants only students who had not taken physics in high school or college. However, we cannot be sure that our high- and low-spatial groups did not differ in conceptual knowledge and, in fact, some of the responses in the protocol study suggested that they did. It is possible that the high-spatial students, as a result of informal education or through interactions with moving objects, had already developed a more sophisticated conceptual understanding of physics concepts and laws, and that this expertise (rather than specific spatial competences) affected their solutions to physics problems. However, it is unlikely that the differences reported in our studies were entirely due to differences in conceptual understanding. First, a factor-analysis study (Kozhevnikov et al., 2002a) has shown that the physics problems studied here load on the same factor as spatial visualization tests, whereas other physics problems load on a separate factor, which was unrelated to spatial visualization ability. If conceptual knowledge of physics was the only relevant variable at play here, we would not expect these physics problems to share common variance with spatial ability tests. It is also interesting to mention that a correlation between spatial ability and kinematics problem solving is no longer present after students receive physics instruction (Kozhevnikov & Thornton, 2006). This suggests that when conceptual knowledge has been already developed, spatial ability is no longer a predictor of their performance on the kinematics problems.

Rather than considering spatial abilities and conceptual knowledge as alternative explanations of the performance differences that we observed in these studies, we speculate that the two factors may be interrelated. That is, high spatial ability may enhance people's ability to gain conceptual knowledge of physics principles in informal situations. Previous studies have shown that high-spatial individuals are better able to perceive and predict complex motions (Law et al., 1993; Isaak & Just, 1995; Kozhevnikov et al., 2002a). Furthermore, the analysis of students' transcripts revealed that, in the case of the frame of reference problem, both low- and high-spatial students started with the same misconception, but the ability of the high-spatial students to maintain different spatial representations appeared to be critical in bringing about a conceptual change. Thus, it is likely that both spatial competence and conceptual understanding are at play. Some basic abilities in spatial processing (e.g., the ability to integrate two components, to decompose images into parts; and to perform spatial transformations) might bring about significant changes in students' conceptual understanding, and this, in turn, might influence students' future strategies and responses. This would also explain why high-spatial students in our studies exhibited more advanced conceptual understanding while attempting extrapolation and graph interpretation problems.

The findings of this research support the idea that spatial ability tests can reflect visual-spatial working memory capacity (Miyake et al., 1991; Shah & Miyake, 1996) in the sense that people who differ in spatial abilities also differ in their ability to solve physics problems that involve multiple spatial parameters. What do multidimensional physics problem solving and spatial visualization ability have in common, and what are other types of physics problems that might require high visual/spatial resources? Both multidimensional physics problems and spatial visualization tasks require the problem solver to simultaneously process multiple pieces of spatial information that tax the supplies of visual/spatial working memory resources. Indeed, all the studies reporting differences in the performance of low- and high-spatial students have examined physics problems with multidimensional spatial parameters (Law et al., 1993; Isaak & Just, 1995; Kozhevnikov, 1999; Kozhevnikov et al., 2002a). Interestingly, however, differences in performance between high- and low-spatial students on these higher dimensional problems disappeared after formal physics instruction with rich visualization technologies (Kozhevnikov & Thornton, 2006). Thus, a curriculum that provides external visualizations via technologies to students who have difficulty generating such visualizations on their own can compensate for such shortcomings.

The finding that spatial ability is related to solving many types of physics problems raises questions about the properties of visual displays that should facilitate the use of visual/spatial processing strategies, particularly to help low-spatial students learn from spatial physics concepts and diagrams. Visualization alone and even dynamic simulations do little to help people understand the dynamics of systems that involve multiple parameters or multiple objects (Kaiser et al., 1992), and low-spatial students in particular have more difficulties than high-spatial students in extracting necessary visual information from animations (Isaak & Just, 1995). However, it has been suggested that animations possessing specific features (e.g., allowing students to break a system down into a point-particle system with a single dynamically relevant parameter or drawing students' attention to a single element of the problem and thus minimizing the influence of other elements) should facilitate the development of spatial understanding and competence (Kaiser et al., 1992). The results from our studies also have some practical applications for the design of visual displays. First, our research suggests that visual simulations should illustrate how velocity vectors change along horizontal and vertical dimensions while an object is moving and demonstrate how these vectors combine to produce an object's overall trajectory. Second, for kinematics graphs, visual simulations should highlight segments of data and associated tick-mark ranges along the axes, rather than the overall shape of the graph, and should lead the learner to analyze and imagine event changes occurring with the subintervals.

Finally, the studies presented here specifically focused on the relationship between spatial ability and kinematics problem solving. Spatial visualization ability should also be useful when solving problems in other physics and science domains. For instance, spatial ability might be especially important in order to visualize invisible phenomena and processes such as electric or magnetic field lines or electric current when solving electricity and magnetism problems. Although some of the implications of our findings should generalize to other domains, further research is needed to explore the specific relationships between visualization skills and students' performance in these domains. The results of this research may reveal important instructional implications for the development and the use of different visualization

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## Notes

1. Only those types of kinematics problems that showed significant correlations with spatial ability tests in the previous research (see Kozhevnikov, 1999; Kozhevnikov et al., 2002a; Kozhevnikov & Thornton, 2006) were included in the current study.
2. Although there is evidence for dissociation between tests of spatial visualization and spatial relations (speeded mental rotation tests) in the psychometric literature (Carroll, 1993), a number of tests of each type must be included in the analysis for these abilities to emerge as separate factors. In the current study, we were not concerned with the dissociation between spatial visualization and spatial relations, so we did not include enough tests for spatial visualization and spatial relations to consider them separately.
3. Parts of the Graph Problem data presented in this study were also included in Kozhevnikov et al. (2002b).

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